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Composition Series in a Group

Def - Let G be a group. A composition series for G is a (finite) chain of successive subgroups of G denoted by

$$\{e\} = G_0 \leq G_1 \leq \cdots \leq G_n = G$$

with the following properties

1.) G_i is a normal subgroup of G_{i+1} for all $0 \leq i \leq n-1$

2.) G_{i+1}/G_i is a simple group for all $0 \leq i \leq n-1$

The length of the composition series is the number n, and the composition factors of the composition series are the quotient groups G_{i+1}/G_i

Note → Simple Group - A group G is said to be a simple group if it has no nontrivial proper normal subgroups. Thus condition (2) requires each of the

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Composition factor G_{i+1}/G_i to have no non-trivial proper normal Subgroups

for example, Consider $G = Z_{12}$ the Cyclic group of Order 12 and write $Z_{12} = \{a\}$ so that $a^{12} = 1$

Recall that every subgroup of a Cyclic group is Cyclic. Thus all of the Subgroups of $G = Z_{12}$ are

$$Z_1 = \{1\} = \langle 1 \rangle$$

$$Z_2 \cong \{1, a^6\} = \langle a^6 \rangle$$

$$Z_3 \cong \{1, a^4, a^6\} = \langle a^4 \rangle$$

$$Z_4 \cong \{1, a^3, a^6, a^9\} = \langle a^3 \rangle$$

$$Z_6 \cong \{1, a^2, a^4, a^6, a^8, a^{10}\} = \langle a^2 \rangle$$

Then Z_1, Z_2, Z_3 and Z_6 are Subgroups of Z_{12}. Since every Cyclic group is abelian.

Each of these Groups are normal Subgroups.

$$\{e\} = Z_1 \leq Z_2 \leq Z_4 \leq Z_{12} = G$$

$$\{e\} = Z_1 \leq Z_2 \leq Z_6 \leq Z_{12} = G$$

$$\{e\} = Z_1 \leq Z_3 \leq Z_6 \leq Z_{12} = G$$

The three series above are all composition series for $G = Z_{12}$.